# CHAPTER 5 Study Guide and Review

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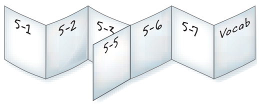


## Key Concepts

### Graphing Quadratic Functions (Lesson 5-1)

- The graph of $y = ax^2 + bx + c$, $a \neq 0$, opens up, and the function has a minimum value when $a > 0$. The graph opens down, and the function has a maximum value when $a < 0$.

### Solving Quadratic Equations (Lessons 5-2 and 5-3)

- The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the $x$-intercepts of its graph.

### Complex Numbers (Lesson 5-4)

- $\boldsymbol{i}$ is the imaginary unit. $\boldsymbol{i}^2 = -1$ and $\boldsymbol{i} = \sqrt{-1}$.

### Solving Quadratic Equations (Lessons 5-5 and 5-6)

- Completing the square: **Step 1** Find one half of $b$, the coefficient of $x$. **Step 2** Square the result in Step 1. **Step 3** Add the result of Step 2 to $x^2 + bx$.
- Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Analyzing Graphs (Lesson 5-7)

- As the values of $h$ and $k$ change, the graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated $|h|$ units left if $h$ is negative or $|h|$ units right if $h$ is positive and $|k|$ units up if $k$ is positive or $|k|$ units down if $k$ is negative.
- Consider the equation $y = a(x - h)^2 + k$, $a \neq 0$. If $a > 0$, the graph opens up; if $a < 0$ the graph opens down. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

## Key Vocabulary

axis of symmetry (p. 237)
completing the square (p. 269)
complex conjugates (p. 263)
complex number (p. 261)
constant term (p. 236)
discriminant (p. 279)
imaginary unit (p. 260)
linear term (p. 236)
maximum value (p. 238)
minimum value (p. 238)
parabola (p. 236)
pure imaginary number (p. 260)
quadratic equation (p. 246)
quadratic function (p. 236)
quadratic inequality (p. 294)
quadratic term (p. 236)
root (p. 246)
square root (p. 259)
vertex (p. 237)
vertex form (p. 286)
zero (p. 246)

## Vocabulary Check

**Choose the term from the list above that best matches each phrase.**

1. the graph of any quadratic function
2. process used to create a perfect square trinomial
3. the line passing through the vertex of a parabola and dividing the parabola into two mirror images
4. a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
5. the solutions of an equation
6. $y = a(x - h)^2 + k$
7. in the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$
8. the square root of $-1$
9. a method used to solve a quadratic equation without using the quadratic formula
10. a number in the form $a + bi$



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# Lesson-by-Lesson Review

## 5–1 Graphing Quadratic Functions (pp. 236–244)

**Complete parts a–c for each quadratic function.**

**a.** **Find the $y$-intercept, the equation of the axis of symmetry, and the $x$-coordinate of the vertex.**

**b.** **Make a table of values that includes the vertex.**

**c.** **Use this information to graph the function.**

**11.** $f(x) = x^2 + 6x + 20$

**12.** $f(x) = x^2 - 8x + 7$

**13.** $f(x) = -2x^2 + 12x - 9$

**14.** **FRAMES** Josefina is making a rectangular picture frame. She has 72 inches of wood to make this frame. What dimensions will produce a picture frame that will frame the greatest area?

**Example 1** **Find the maximum or minimum value of $f(x) = -x^2 + 4x - 12$.**

Since $a < 0$, the graph opens down and the function has a maximum value. The maximum value of the function is the $y$-coordinate of the vertex. The $x$-coordinate of the vertex is $x = \frac{-4}{2(-1)}$ or 2. Find the $y$-coordinate by evaluating the function for $x = 2$.

$f(x) = -x^2 + 4x - 12$ Original function

$f(2) = -(2)^2 + 4(2) - 12$ Replace $x$ with 2.

or $-8$

Therefore, the maximum value of the function is $-8$.

## 5–2 Solving Quadratic Equations by Graphing (pp. 246–251)

**Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

**15.** $x^2 - 36 = 0$

**16.** $-x^2 - 3x + 10 = 0$

**17.** $-3x^2 - 6x - 2 = 0$

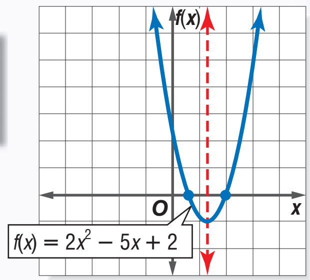
**18.** $\frac{1}{5}(x + 3)^2 - 5 = 0$

**19.** **BASEBALL** A baseball is hit upward at 100 feet per second. Use the formula $h(t) = v_ot - 16t^2$, where $h(t)$ is the height of an object in feet, $v_o$ is the object's initial velocity in feet per second, and $t$ is the time in seconds. Ignoring the height of the ball when it was hit, how long does it take for the ball to hit the ground?

**Example 2** **Solve $2x^2 - 5x + 2 = 0$ by graphing.**

The equation of the axis of symmetry is $x = -\frac{-5}{2(2)}$ or $x = \frac{5}{4}$.

| $x$ | 0 | $\frac{1}{2}$ | $\frac{5}{4}$ | 2 | $\frac{5}{2}$ |
|---|---|---|---|---|---|
| $f(x)$ | 2 | 0 | $-\frac{9}{8}$ | 0 | 2 |




The zeros of the related function are $\frac{1}{2}$ and 2. Therefore, the solutions of the equation are $\frac{1}{2}$ and 2.

# Study Guide and Review

## 5-3 Solving Quadratic Equations by Factoring (pp. 253–258)

Write a quadratic equation in standard form with the given root(s).

**20.** $-4, -25$ **21.** $10, -7$ **22.** $\frac{1}{3}, 2$

Solve each equation by factoring.

**23.** $x^2 - 4x - 32 = 0$ **24.** $3x^2 + 6x + 3 = 0$

**25.** $5y^2 = 80$ **26.** $25x^2 - 30x = -9$

**27.** $6x^2 + 7x = 3$ **28.** $2c^2 + 18c - 44 = 0$

**29. TRIANGLES** Find the dimensions of a triangle if the base is $\frac{2}{3}$ the length of the height and the area is 12 square centimeters.

**Example 3** Write a quadratic equation in standard form with the roots 3 and −5.

$(x - p)(x - q) = 0$ Write the pattern.

$(x - 3)(x + 5) = 0$ $p = 3$ and $q = -5$

$x^2 + 2x - 15 = 0$ Use FOIL.

**Example 4** Solve $x^2 + 9x + 20 = 0$ by factoring.

$x^2 + 9x + 20 = 0$ Original equation

$(x + 4)(x + 5) = 0$ Factor the trinomial.

$x + 4 = 0$ or $x + 5 = 0$ Zero Product Property

$x = -4$ $x = -5$

The solution set is $\{-5, -4\}$.

## 5-4 Complex Numbers (pp. 259–266)

Simplify.

**30.** $\sqrt{45}$ **31.** $\sqrt{64n^3}$

**32.** $\sqrt{-64m^{12}}$

**33.** $(7 - 4i) - (-3 + 6i)$

**34.** $(3 + 4i)(5 - 2i)$ **35.** $(\sqrt{6} + i)(\sqrt{6} - i)$

**36.** $\frac{1 + i}{1 - i}$ **37.** $\frac{4 - 3i}{1 + 2i}$

**38. ELECTRICITY** The impedance in one part of a series circuit is $2 + 3j$ ohms, and the impedance in the other part of the circuit is $4 - 2j$. Add these complex numbers to find the total impedance in the circuit.

**Example 5** Simplify $(15 - 2i) + (-11 + 5i)$.

$(15 - 2i) + (-11 + 5i)$

$= [15 + (-11)] + (-2 + 5)i$ Group the real and imaginary parts.

$= 4 + 3i$ Add.

**Example 6** Simplify $\frac{7i}{2 + 3i}$.

$\frac{7i}{2 + 3i} = \frac{7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$ $2 + 3i$ and $2 - 3i$ are conjugates.

$= \frac{14i - 21i^2}{4 - 9i^2}$ Multiply.

$= \frac{21 + 14i}{13}$ or $\frac{21}{13} + \frac{14}{13}i$ $i^2 = 1$

**Mixed Problem Solving**
For mixed problem-solving practice, see page 930.

## 5–5 Completing the Square (pp. 268–275)

**Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.**

**39.** $x^2 + 34x + c$ **40.** $x^2 - 11x + c$

**Solve each equation by completing the square.**

**41.** $2x^2 - 7x - 15 = 0$

**42.** $2x^2 - 5x + 7 = 3$

**43. GARDENING** Antoinette has a rectangular rose garden with the length 8 feet longer than the width. If the area of her rose garden is 128 square feet, find the dimensions of the garden.

**Example 7** **Solve $x^2 + 10x - 39 = 0$ by completing the square.**

$$x^2 + 10x - 39 = 0$$
$$x^2 + 10x = 39$$
$$x^2 + 10x + 25 = 39 + 25$$
$$(x + 5)^2 = 64$$
$$x + 5 = \pm 8$$
$$x + 5 = 8 \text{ or } x + 5 = -8$$
$$x = 3 \qquad x = -13$$

The solution set is $\{-13, 3\}$.

## 5–6 The Quadratic Formula and the Discriminant (pp. 276–283)

**Complete parts a–c for each quadratic equation.**

**a.** Find the value of the discriminant.
**b.** Describe the number and type of roots.
**c.** Find the exact solutions by using the Quadratic Formula.

**44.** $x^2 + 2x + 7 = 0$

**45.** $-2x^2 + 12x - 5 = 0$

**46.** $3x^2 + 7x - 2 = 0$

**47. FOOTBALL** The path of a football thrown across a field is given by the equation $y = -0.005x^2 + x + 5$, where $x$ represents the distance, in feet, the ball has traveled horizontally and $y$ represents the height, in feet, of the ball above ground level. About how far has the ball traveled horizontally when it returns to the ground?

**Example 8** **Solve $x^2 - 5x - 66 = 0$ by using the Quadratic Formula.**

In $x^2 - 5x - 66 = 0$, $a = 1$, $b = -5$, and $c = -66$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)}$$

$$= \frac{5 \pm 17}{2}$$

Write as two equations.

$$x = \frac{5 + 17}{2} \text{ or } x = \frac{5 - 17}{2}$$
$$= 11 \qquad = -6$$

The solution set is $\{-6, 11\}$.

# 5 Study Guide and Review

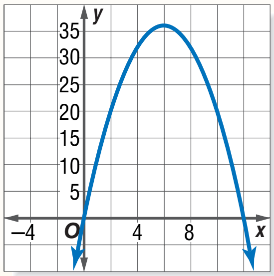
## 5–7 Analyzing Graphs of Quadratic Functions (pp. 286–292)

**Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function.**

**48.** $y = -6(x + 2)^2 + 3$ **49.** $y = -\frac{1}{3}x^2 + 8x$

**50.** $y = (x - 2)^2 - 2$ **51.** $y = 2x^2 + 8x + 10$

**52. NUMBER THEORY** The graph shows the product of two numbers with a sum of 12. Find an equation that models this product and use it to determine the two numbers that would give a maximum product.




**Example 9** **Write the quadratic function $y = 3x^2 + 42x + 142$ in vertex form. Then identify the vertex, axis of symmetry, and the direction of opening.**

$y = 3x^2 + 42x + 142$ Original equation

$y = 3(x^2 + 14x) + 142$ Group and factor.

$y = 3(x^2 + 14x + 49) + 142 - 3(49)$ Complete the square.

$y = 3(x + 7)^2 - 5$ Write $x^2 + 14x + 49$ as a perfect square.

So, $a = 3$, $h = -7$, and $k = -5$. The vertex is at $(-7, -5)$, and the axis of symmetry is $x = -7$. Since $a$ is positive, the graph opens up.

## 5–8 Graphing and Solving Quadratic Inequalities (pp. 294–301)

**Graph each inequality.**

**53.** $y > x^2 - 5x + 15$ **54.** $y \geq -x^2 + 7x - 11$

**Solve each inequality using a graph, a table, or algebraically.**

**55.** $6x^2 + 5x > 4$ **56.** $8x + x^2 \geq -16$

**57.** $4x^2 - 9 \leq -4x$ **58.** $3x^2 - 5 > 6x$

**59. GAS MILEAGE** The gas mileage $y$ in miles per gallon for a particular vehicle is given by the equation $y = 10 + 0.9x - 0.01x^2$, where $x$ is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon.

**Example 10** **Solve $x^2 + 3x - 10 < 0$.**

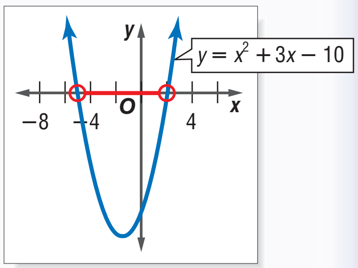
Find the roots of the related equation.

$0 = x^2 + 3x - 10$ Related equation

$0 = (x + 5)(x - 2)$ Factor.

$x + 5 = 0$ or $x - 2 = 0$ Zero Product Property

$x = -5$ $x = 2$ Solve each equation.




The graph opens up since $a > 0$. The graph lies below the $x$-axis between $x = -5$ and $x = 2$. The solution set is $\{x \mid -5 < x < 2\}$.